

The Rousdon Variable Star Observations.
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1. The observations of long-period variables made during the years 1886–1900 at the Rousdon Observatory, Lyme Regis, Devon, under the direction of the late Sir C. E. Peek, Bart., were put into my hands for discussion and publication after Sir C. E. Peek's death, and will appear as Vol. LV. of the *Memoirs*. A brief account of the discussion may be convenient for other observers.

2. Comparisons were made whenever possible between the variable and *five* stars within a field approximately 1° square. It was of course impossible always to find five stars in this area within a few tenths of a magnitude of the variable; and the differences estimated often exceed a magnitude, sometimes two or three. The accidental errors of such comparisons are naturally large; but there are, nevertheless, distinct advantages in the method which appear on discussion. As regards systematic error it appears that a certain quantity (about 0.2 mag.) must be *numerically subtracted* from all estimated differences greater than about ± 0.4 mag. But the value of this quantity changes sensibly with the brightness of the stars compared, and also at two epochs during the work. Still it is not difficult to correct with tolerable accuracy for this arbitrary scale.

3. There is some evidence of systematic error depending on position-angle, but it is conflicting; and, after several vain attempts to establish laws for it from the material available, the quest was abandoned.

4. There seems to be no doubt that the magnitudes for the comparison stars determined at Harvard will not in all cases suit the Rousdon observations. Mr. Grover sees some of them systematically brighter or fainter, doubtless owing to peculiarities of colour. Some trouble was taken to deduce individual magnitudes which would suit his observations, while yet conforming generally to the Harvard scale. But the consequent corrections have been exhibited in a separate column (as also those for the scale value) so that they can be omitted if this is thought desirable.

5. A comparison with the Harvard observations (*Harvard Annals*, vol. xxxvii., Part I.) is given. Two important deductions are made from this comparison.

(a) *Systematic Errors*.—The stars observed were divided into two classes, A and B, according to their “redness” as given in Chandler's Third Catalogue. Six stars in Class A (viz. *R*, *S*, and *T Cassiopeiae*, *S* and *T Cephei*, and *R Aurigae*) have a mean redness 7.1 ; and five stars in Class B (viz. *S* and *T Ursae Majoris*, *S Boötis*, *R Camelopardi*, and *R Draconis*) have a mean redness 2.4 . The differences, Rousdon—Harvard, were

arranged according to the mean Rousdon magnitude, and gave the following mean results :

Rousdon Mag.	...	6.5	7.5	8.5	9.5	10.5	11.5	12.5
	Redness.							
Class A	...	7.1	+ .6	+ .3	- .1	- .3	- .6	- .7
Class B	...	2.4	...	0	0	0	- .1	0

There is thus a progressive difference for *red stars* amounting to 0.3 per magnitude ! This personal equation far exceeds any other systematic error discussed in the *Memoir*, and seems to demand the most careful investigation if series of observations by different observers are to be combined. It throws some light on the special results for comparison stars mentioned in the last paragraph. It does not affect discussions of observations by the same observer unless he is found to change his habit ; but it seems quite possible that there may be such changes of habit, and a series of comparisons between observers at different times would give valuable information.

T Cephei. Assumed
Residuals for each.

Group.	Julian Day.	1	2	3	4	5
		0	32	64	96	128
1871
2255	...	- .3	- .2	- .5	+ .2	+ .2
2639	...	- .3	- .6	- .5	+ .3	+ .3
3023	...	+ .4	- .3	+ .1	- .1	+ .5
3407	...	+ .4	+ .7	+ .5	- .1	+ .5
3791	...	+ .6	+ .2	+ .7	+ .6	+ .2
4175	...	+ .5	+ 1.0	+ .1	+ .1	(- .2)
4559	...	- .5	- .1	- .3	- .7	- .8
4943	...	- .2	+ .6	.0
Mean	...	+ 0.08	+ 0.16	+ 0.01	+ 0.04	+ 0.10
Mag. from Curve	...	7.50	6.80	6.38	6.83	7.56
2255 + (2)	...	- .2	- 1.1	- .9	+ .4	+ 1.0
3791 + (2)	...	+ .6	+ 1.1	+ .5	.0	- .8
Difference	...	+ .8	+ 2.2	+ 1.4	- .4	- 1.8
Factor 5.09	...	+ 4.1	+ 11.2	+ 7.1	- 2.0	- 9.2
p - f	...	+ 0.6	+ 1.2	+ 0.1	- 1.3	- 1.4
Corr. Period	...	+ 6.8	+ 9.3	...	+ 1.5	+ 6.6

(b) *Accidental Errors*.—There are occasions when an observer records a sensible deviation from a mean curve, amounting to more than half a magnitude. It is very important to know whether these deviations are real or accidental. Observers themselves sometimes feel convinced of their reality. It is interesting to find that, in the case of *T Cassiopeiae*, at any rate, which is discussed in detail in the *Memoir*, when a sensible deviation of this kind is observed at Rousdon it is not confirmed at Harvard, and *vice versa*. Systematic difference was allowed for before making the comparison. From this and other evidence it would seem that the variation of light in these stars is possibly a strictly periodic phenomenon, though several waves may be superposed.

6. Besides tables giving the observations for each night separately, compendious tables have been formed summarising the results. The following table for *T Cephei* will sufficiently illustrate the method adopted :

period, 384 days.

Group of 32 Days.

6 160	7 192	8 224	9 256	10 288	11 320	12 352
...	+ 1.1	+ .1	.1	— .3	— .5	— .1
+ .3	+ .5	+ .2	+ .6	— .1	.0	+ .2
.0	(— .1)	— .2	— .5	— .6	.0	— .4
.0	— .2	+ .3	+ .7	— .2	— .1	+ .3
+ .1	— .4	+ .4	— .1	— .1	+ .1	+ .5
+ .1	— .2	(.0)	+ .2	+ .4	+ .4	+ .6
— .5	— .8	— 1.1	— .3	— .1	— .1	— .6
— .6	— 1.0	— .7	— .4	— .1	+ .2	— .7
...
— 0.09	— 0.14	— 0.12	+ 0.03	— 0.14	0.00	— 0.03
8.32	9.07	9.62	9.53	8.93	8.10	7.60
+ .3	+ .2	+ .3	+ .8	— .9	— .1	+ .1
— 1.0	— 2.0	— 1.8	— .5	+ .2	+ .5	— .7
— 1.3	— 2.2	— 2.1	— 1.3	+ 1.1	+ .6	— .8
— 6.6	— 11.2	— 10.7	— 6.6	+ 6.0	+ 3.1	— 4.1
— 1.3	— 1.2	— 0.6	+ 0.7	+ 1.5	+ 1.2	+ 0.6
+ 5.1	+ 9.3	+ 17.8	— 9.4	+ 4.0	+ 2.6	— 6.8

7. Starting from an arbitrary date with an assumed period the observations were divided into groups of one-twelfth period and a mean light curve formed as shown in the line "Mag. from Curve." The individual observations were compared with this curve and the residuals for each group combined. The mean date to which any residual in the table corresponds is formed by adding the numbers at the top of the column and the end of the row and the omitted constant 2410000 to get the Julian date. Thus the first residual +1.1 corresponds to Julian date 2410000 + 1871 + 192 = 2412063. From the means of the columns it is seen that the assumed light curve is not quite correct, but the corrections indicated, or any portion of them, can readily be applied.

8. In the lower part of the table it is shown how a correction to the assumed period may readily be deduced. Sums of residuals for the period 2255 and two following are denoted by 2255 + (2). Comparing these with a later series, the differences are negative from maximum to minimum, and positive from minimum to maximum; indicating that the assumed period is too short. Applying the "factor 5.09" and the divisor " $p - f$ " we get a correction to the period from each column, as fully explained in the memoir. The weighted mean of these corrections for *T Cephei* is +4.9 days, so that the true period deduced from the Rousdon observations is 388.9 days.

9. In this way an accurate value for the period has been deduced for each star from the Rousdon observations alone; and compared with that given in Chandler's Third Catalogue (*A. J.* No. 379). His revised "Elements" (*A. J.* No. 553) have since been received and compared with the Rousdon determinations in a postscript. The effect of the revision is so marked that the following summary of it may be repeated here. For the majority of the stars the period is variable according to a definite law, that for *S Cassiopeiæ*, for instance, being denoted by

$$610.5 \text{ E} + 37 \sin (15^\circ \text{ E} + 59^\circ)$$

the mean period of 610.5 days being liable to regular variations of thirty-seven days each way. For comparison with Rousdon, the periods have been obtained from the formulæ by substituting the values of E appropriate to the Rousdon observations.

No.	Star's Name.	Chandler's Period.		Rousdon Period.	R—Old.	R—New.
		Old. d	New. d			
107	T Cassiopeiæ	445.0	443.5	444.4	— 0.6	+ 0.9
432	S Cassiopeiæ	606.2	619.4	630.7	+ 24.5	+ 11.3
1855	R Aurigæ	460.2	455.0	454.2	— 6.0	— 0.8
2100	U Orionis	375	375	374.4	— 0.6	— 0.6
2478	R Lyncis	380.0	377.4	377.8	— 2.2	+ 0.4
3825	R Ursæ Majoris	304.2	302.4	300.4	— 3.8	— 2.0

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No.	Star's Name.	Chandler's Period.		Rousdon Period.	R—Old.		R—New.
		Old. d	New. d		d		d
4511	T Ursæ Majoris	254·8	254·8	255·5	+ 0·7		+ 0·7
4557	S Ursæ Majoris	223·0	224·4	224·5	+ 1·5		+ 0·1
5157	S Boötis	259·8	266·5	265·9	+ 6·1		— 0·6
5190	R Camelopardi	269·5	273·9	273·3	+ 3·8		— 0·6
5504	S Coronæ	360·8	362·6	363·3	+ 2·5		+ 0·7
5955	R Draconis	245·6	245·6	245·8	+ 0·2		+ 0·2
6044	S Herculis	308·1	305·7	309·0	+ 0·9		+ 3·3
6449	T Draconis	(569)	426	428·3	...		+ 2·3
7045	R Cygni	425·7	426·6	427·0	+ 1·3		+ 0·4
7120	χ Cygni	406·2	406·2	407·0	+ 0·8		+ 0·8
7220	S Cygni	320·8	324·1	327·0	+ 6·2		+ 2·9
7609	T Cephei	387	387	388·9	+ 1·9		+ 1·9
7779	S Cephei	484	487·9	489·8	+ 5·8		+ 1·9
8600	R Cassiopeiæ	424·7	436·5	438·7	+ 14·0		+ 2·2

10. It will be seen that the new formulæ suit the Rousdon results much better than the old in every case but that of *S Herculis*. For *S Cassiopeiæ* the period is still capable of improvement; and perhaps also for *R Ursæ Majoris* and *R Cassiopeiæ*. But the general agreement is such as to warrant considerable confidence in Chandler's formulæ.

11. He gives no definite periods for *S Persei* or *R Ursæ Minoris*. For *S Persei* a main period of 840 days, and a subsidiary one of 1120 days are indicated by the Rousdon observations; though there are also traces of a much longer period (3360 days?). For *R Ursæ Minoris* the Rousdon observations indicate 320 days.

12. Some attempt is made in the Memoir to consider the possible origin of Chandler's periodic terms. The various light curves were analysed harmonically, and they seemed to fall into a series, the coefficients of the separate harmonics being related to one another. Thus dividing the stars into five groups, according to the value of A in the formula

$$A \sin \theta + B \cos \theta + C \sin 2\theta + D \cos 2\theta + E \sin 3\theta + F \cos 3\theta$$

where θ is reckoned from maximum, and the whole range of variation is represented by 6, we get the following values for the coefficients:

Group.	A	B	C	D	E	F
I.	+ ·83	— 2·50	— ·49	— ·14	+ ·16	— ·19
II.	+ ·40	— 2·78	— ·22	+ ·27	— ·09	— ·20
III.	— ·05	— 2·86	+ ·01	+ ·04	— ·02	— ·13
IV.	— ·77	— 2·70	+ ·30	— ·23	— ·03	— ·04
V.	— 1·19	— 2·56	+ ·33	— ·36	+ ·09	— ·04

13. It is only natural that B should vary with A, since without the other terms, which are small, the range would be $2(A^2 + B^2)^{\frac{1}{2}}$, and we have made the range constant. But there is less reason for the obvious run in the other terms. As regards D, the apparent anomaly in the value for Group I. is removed when we put the expression into the alternative form

$$H_1 \cos (\theta + G_1) + H_2 \cos (2\theta + G_2) + H_3 \cos (3\theta + G_3)$$

when the negative sign for D is seen to be merely a consequence of the continuous change in G_2 . The following approximate formulæ are merely illustrative, to show roughly the dependence of all these quantities on a single variable, and have obviously no physical significance :

$$H_2 = 0.5 A^{\frac{2}{3}} \quad H_3 = -0.14 - 0.20 A \quad G_2 = 130^\circ A$$

$$G_3 = 80^\circ A - 40^\circ$$

and the success with which these formulæ represent the observed quantities is shown in the following table :

Group.	A	H ₂	G ₂	O		D		H ₃	G ₃	E		F	
				Calc.	Obs.	Calc.	Obs.			Calc.	Obs.	Calc.	Obs.
I.	+ .83	+ .44	+ 108	-.42	-.49	-.14	-.14	-.31	+ 26	+ .13	+ .16	-.27	-.19
II.	+ .40	+ .27	+ 52	-.21	-.22	+ .17	+ .27	-.22	- 8	-.03	-.09	-.22	-.20
III.	- .05	+ .06	- 6	+ .01	+ .01	+ .06	+ .04	-.13	- 44	-.09	-.02	-.09	-.13
IV.	- .77	+ .42	- 100	+ .41	+ .30	-.07	-.23	+ .01	-.02	+ .01	-.03	.00	-.04
V.	- 1.19	+ .56	- 155	+ .24	+ .33	-.51	-.36	+ .10	- 135	+ .07	+ .09	-.07	-.04

14. If relations of this kind can be established among the harmonic coefficients, we may be able to separate out two classes of periodic terms, one set following laws similar to those sketched above, and another with smaller amplitude and slightly different period which, by interference with the main set, cause the oscillation in epoch of maximum which Chandler represents by such expressions as

$$610.5 E + 37 \sin (15^\circ E + 59^\circ)$$

At any rate a promising line of investigation seems to open here. It was my intention to collect more material so as to improve the formulæ ; and I made a preliminary search for well-established light curves published by other observers in order to submit them to harmonic analysis. But I had little success in finding them—perhaps through lack of acquaintance with the literature of the subject.

15. *I should be particularly obliged if observers who have formed well-determined light curves would kindly let me have copies of them or a reference to the publication in which they are given.*

16. While writing this note it occurred to me that possibly the sun-spot curve might exhibit features either resembling those

of the variable stars or obviously differing from them. Seen as a star, the variation in total light of the Sun, owing to its variations in spottedness, is no doubt extremely small, perhaps quite insensible; but the features of the light curves above studied seem to have little or no relation to the total range of variation, and may persist even when this has become so small as in the case of our Sun. To test the point, Wolf's sun-spot numbers for the forty-four years 1849-1892 were tabulated in four periods of eleven years,* and the sums taken as follows:

1	2	3	4	5	6	7	8	9	10	11
362	309	253	195	128	77	44	24	61	196	354

It was not difficult to draw a regular curve through these points and to read off the ordinates at twelve epochs for harmonic analysis.

17. But now came the question, Is the Sun to be regarded as (A) slightly brighter at a sunspot maximum, or (B) slightly fainter? If spots mean loss of light (B) is the proper supposition; but since they also mean increase of activity, it would be more natural to take (A). Both hypotheses were therefore tried, with the results given below.

Hypothesis A. Sun brighter when spotted.

18. Twelve equidistant ordinates in fractions of the greatest as unity, counting from sun-spot maximum as zero.

Max.	30°.	60°.	90°.	120°.	150°.	180°.	210°.	240°.	270°.	300°.	330°.
·00	·09	·23	·38	·54	·71	·84	·94	·99	·93	·65	·22

Analysing these by the same process as that used for the variables, we get:

	A.	B.	C.	D.	E.	F.	H ₁ .	G ₂ .	H ₃ .	G ₃ .
Observed	-1·49	-2·32	+1·11	-·70	+·19	-·19	+·71	-171°	+·27	-135°
Formulae	(-1·49)	-2·29	-·16	-·64	+·05	-·15	+·65	-194	+·16	-160

Below the "observed" values are given the values which would be deduced from the variable star formulæ given above by subtracting the value -1·49 for A. The agreement is good, the chief discrepancy being that in C, which would be removed by a comparatively slight change in the epoch G₂.

A slight change either in A or in the formula $G_2 = 130^\circ$ A would make a satisfactory adjustment. Considering that the value of A for the Sun is much larger than in any of the series from which the formulæ were deduced, so that we are extrapolating for the Sun, the accordance may be considered satisfactory.

* For our present purpose a slight error in the period does not matter.

Hypothesis B. Sun fainter when spotted.

19. Twelve equidistant ordinates as before, but counting from sun-spot minimum as zero.

Max.	30°.	60°.	90°.	120°.	150°.	180°.	210°.	240°.	270°.	300°.	330°.
·00	·10	·42	·85	1·00	·90	·74	·60	·43	·26	·13	·05

Analysing which we get :

	A.	B.	C.	D.	E.	F.	H ₂ .	G ₂ .	H ₃ .	G ₃ .
Observed	+1·51	-2·35	·46	·54	·23	+·14	+·72	+138	·27	+236
Formulae (+1·51)	-1·67	+·17	·63	+·43	·08	+·65	+195	·44	+ 80	

The difficulties of reconciling the observed quantities with the formulæ are now more serious. The H₃ agree as well as before ; but the discrepancy in G₂ is doubled, that in G₃ is as large as it can well be, and there is a serious discrepancy in B as calculated from the formula $B^2 = 8·20 - 1·8 (A + 0·20)^2$, deduced from the variable star results. Without a considerable modification of the formulæ, the sun-spot curve will not fit in with this end of the series at all.

20. We must not lay too much stress on the evidence, but so far as it goes it supports the conclusions :

(1) That the sun-spot curve falls into the series of variable-star curves, but outside the series presented by the Rousdon variables.

(2) That a maximum of sun-spot activity corresponds to a maximum of a variable star.

The Rotation Period of the Planet Saturn. By G. W. Hough.

In the *Monthly Notices* for January 1904, Mr. W. F. Denning suggests a wrong identification of the spots on *Saturn* as given in *Monthly Notices* for December, 1903. As so little is known regarding the time of rotation of the planet, the question is one of considerable importance. Of the spots observed on June 23-27 and July 13-29 there can be no question whatever. The latitudes agree very closely, and the spot was then conspicuous. And it was the only well-defined and conspicuous spot seen on the planet during the opposition. The spot observed by Burnham on August 19 was faint, and appeared to be out in latitude, and for these reasons its identity with the original spot may be questioned.

I think it highly probable, however, that previous to the middle of August the Barnard spot had lost its outline and ceased to be a well-defined object. The small spot observed on July 14, according to Barnard's micrometer measure, followed